Première réunion générale du GDR NBODY : Problème Quantique à N Corps en Chimie et Physique

Marcella Grasso

Excitations of correlated nucleons within the second random-phase approximation

IJCLab

IPN INSTITUT DE PHYSIQUE NUCLÉAIRE ORSAY

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université PARIS-SACLAY

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A few words on the context… nuclear energy-density- functional (EDF) theories for the nuclear many-body problem

Functionals for nuclear physics (nuclei and infinite matter). Empirical way

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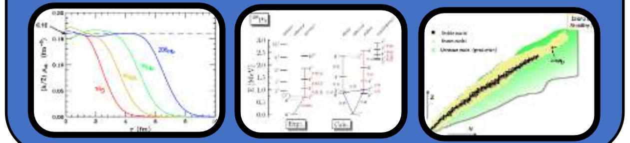
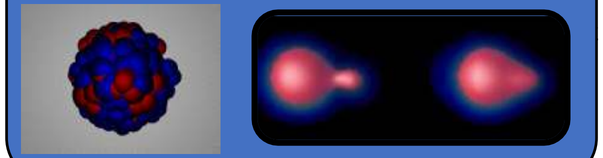
GROUND STATE-STRUCTURE OF THE ATOMIC NUCLEUS

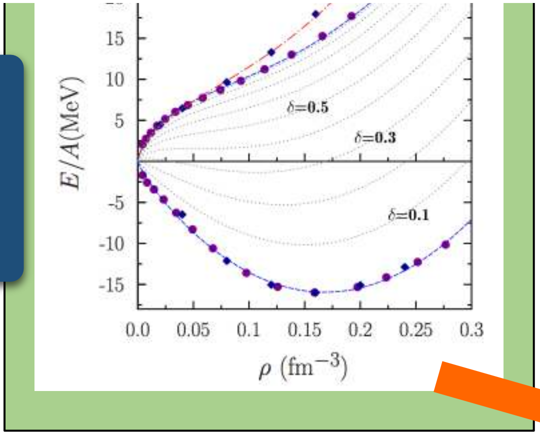


SMALL AND LARGE AMPLITUDE DYNAMICS



NUCLEAR ASTROPHYSICS

Equation of state
of infinite matter (ideal infinite system composed by neutrons and protons)



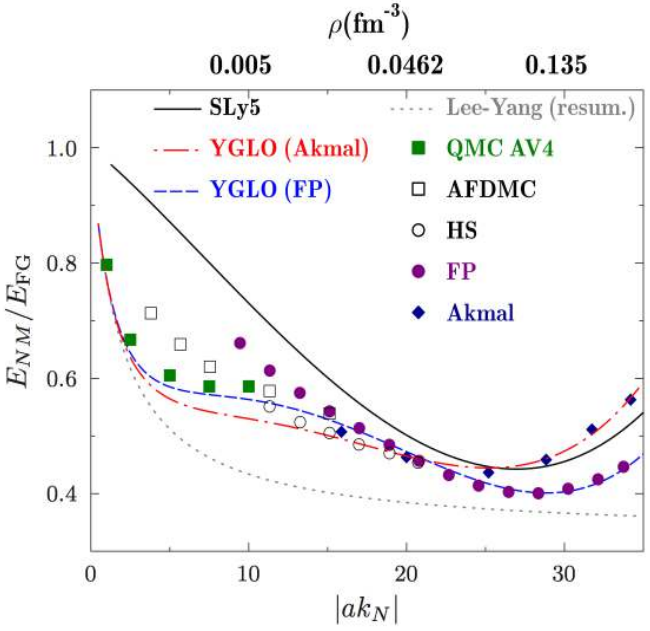
May constrain (fits of parameters)/**guide to construct**

Functionals/interactions …

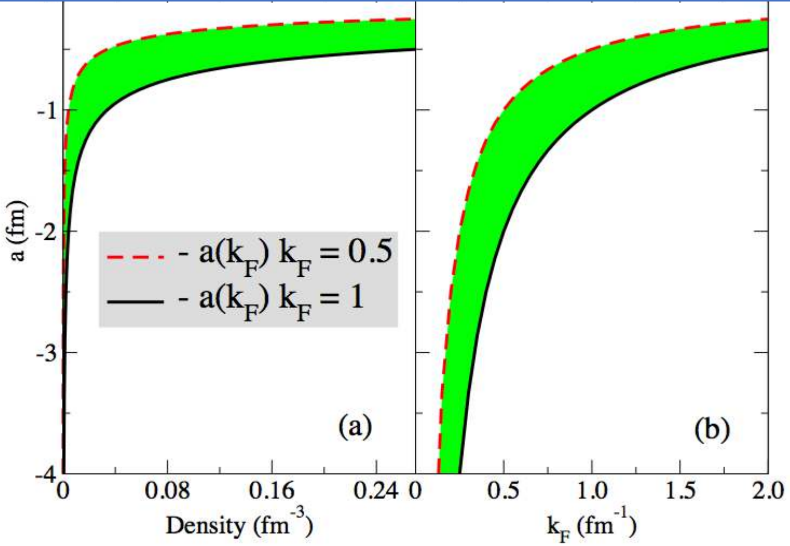
Modern generation of (EFT-guided/inspired and/or ab-initio benchmarked) nuclear functionals

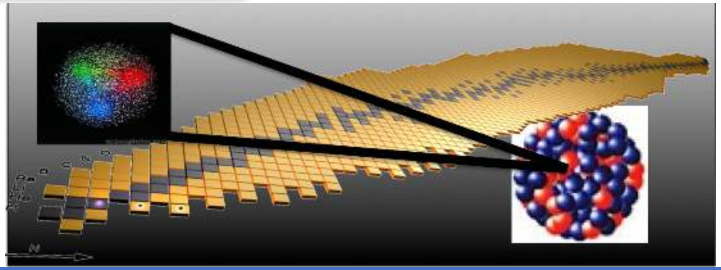
Bridging with EFT/ab initio (reducing the empirical character)

Correctly describing low-density Fermi gases For neutron matter (resummation techniques)

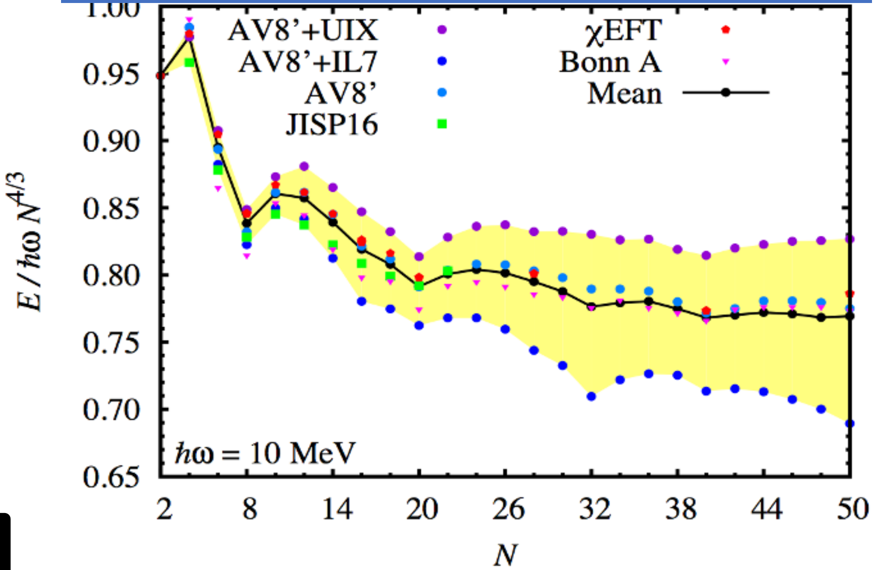


Tuning the nn s-wave scattering length for describing all density scales and the unitary limit



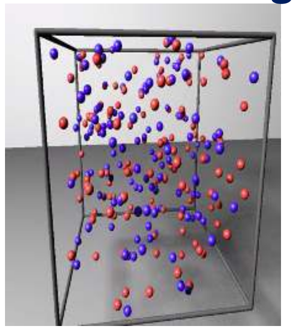


Benchmarking with ab-initio models in neutron drops

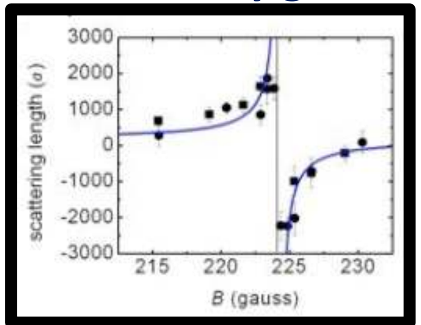


- Moghrabi et al, PRL 105, 262501 (2010)
- Yang et al, PRC 94, 034311 (2016)
- Grasso et al, Phys. Scr. 91, 6 (2016)
- Yang et al, PRC 94, 031301(R) (2016)
- Lacroix, PRA 94, 043614 (2016)
- Lacroix et al, PRC 95, 054306 (2017)
- Yang et al, PRC 95 , 054325 (2017)

From a dilute gas



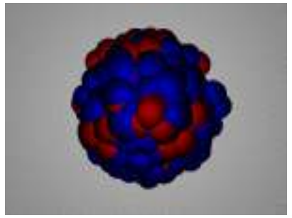
To a unitary gas



- Grasso et al, PRC 95, 054327 (2017)
- Yang et al, PRC 96, 034318 (2017)
- Boulet et al, PRC 97, 014301 (2018)
- Bonnard et al, PRC 98, 034319 (2018)
- Grasso, Prog. Part. Nucl. Phys. 106, 256 (2019)

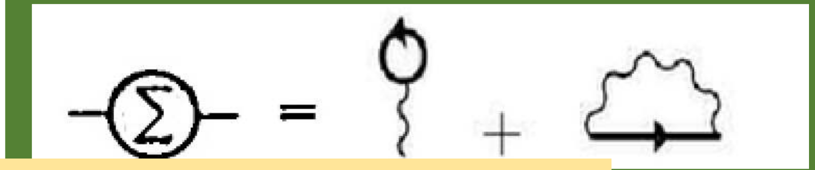
Outline

- Higher-order RPA with the second RPA (SRPA) model (beyond mean field) within the EDF framework



Describing excitation spectra in a many-
b

Mean-field proper self-energy



MEAN FIELD : CANNOT PROPERLY DESCRIBE WIDTHS (DAMPING) AND FRAGMENTATION OF EXCITED STATES

ns
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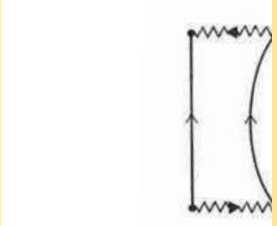
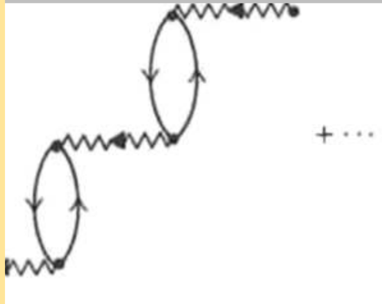
A)

Ring contribut



d (effective
interaction



Bridges with C
Cluster Theory

- Scuseria et al, J.
- Scuseria et al, J. Chem. Phys. 139, 104113 (2013)

Beyond mean field: SRPA model : formally established in the 60s

$$Q_\nu^\dagger = \sum_{ph} (X^\nu_{ph} a_p^\dagger a_h - Y^\nu_{ph} a_h^\dagger a_p) + \sum_{p<p',h<h'} (X^\nu_{php'h'} a_p^\dagger a_h a_{p'}^\dagger a_{h'} - Y^\nu_{php'h'} a_h^\dagger a_p a_{h'}^\dagger a_{p'})$$

Excitation operators: 2p2h configurations are included, together with the RPA 1p1h configurations

Different formal derivations
- **Variational procedure**
- **Equations of motion method**
- **Small-amplitude limit of the time-dependent density matrix method**

- Sawicki, Phys. Rev. 126, 2231 (1962)
- da Providencia, Nucl. Phys. 61, 87 (1965)
- Yannouleas, Phys. Rev. C 35, 1159 (1987)
- Tohyama, Gong, Z. Phys.A 332, 269 (1989)

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B}^* & -\mathcal{A}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}^\nu \\ \mathcal{Y}^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}^\nu \\ \mathcal{Y}^\nu \end{pmatrix}$$

Same compact form as RPA equations

$$\mathcal{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix},$$

$$\mathcal{X}^\nu = \begin{pmatrix} X_1^\nu \\ X_2^\nu \end{pmatrix}, \quad \mathcal{Y}^\nu = \begin{pmatrix} Y_1^\nu \\ Y_2^\nu \end{pmatrix}.$$

A_{11} and B_{11}: standard RPA matrices

A_{12}, A_{21}, B_{12}, and B_{21}: coupling between 1p1h and 2p2h

A_{22} and B_{22}: 2p2h sector

1 and 2:

short-hand notation for 1p1h and 2p2h

Born in the 60s

1970 -> 2005

Last decade

Unaffordable computational effort (number of 2p2h configurations and required number of eigenvalues

Computation of fragmentation and spreading width:
strong cuts in the 2p2h space, Second Tamm-Dancoff, truncations and ap

No approximations/cuts in 2p2h sector and large 2p2h cutoff values

First MODERN CALCULATIONS 2009-2012 (first step: a not too heavy nucleus: ^{16}O):

Microscopic interaction (derived from Argonne V18)

- Papakonstantinou and Roth, Phys. Lett. B 671, 356 (2009)
- Papakonstantinou and Roth, Phys. Rev. C 81, 024317 (2010)

Phenomenological Skyrme and Gogny interactions

- Gambacurta, Grasso, and Catara, Phys. Rev. C 81, 054312 (2010)
- Gambacurta, Grasso, and Catara, J. Phys. G 38, 035103 (2011)
- Gambacurta, Grasso, and Catara, Phys. Rev. C 84, 034301 (2011)
- Gambacurta, Grasso, De Donno, Co’, and Catara, Phys. Rev. C 86, 021304(R) (2012)

RPA and SRPA stability

- Tselyaev, PRC 88, 054301 (2013) (subtraction)
- Papakonstantinou, PRC 90, 024305 (2014) (correlated ground state)

- **Stability Condition (Thouless 1960)**

HF state minimizes the expectation value of the Hamiltonian -> RPA stability matrix is positive semi-definite (real eigenvalues; eigenvectors with positive eigenvalues have positive norm)

$$\cancel{S^{RPA} = M^{RPA} \Omega^{RPA}} = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}$$

$$M^{RPA} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- HF state minimizes the expectation value of the Hamiltonian -> does not guarantee that the SRPA stability matrix is positive semi-definite (instabilities and unphysical strong shifts of the spectrum)

IN ADDITION TO STABILITY PROBLEMS

In the EDF scheme for beyond mean field:
- Ultraviolet divergence for zero-range interactions (dependence on the energy cutoff for the 2p2h configurations)
– Overcounting of correlations (if parameters of the interactions are adjusted at the mean-field level)

EDF: double counting and requiring that the SRPA stability matrix is positive semi-definite (Tselyaev)

- Correlations implicitly included in the functional (because of the adjustment of the parameters at mean-field level to reproduce some observables). In the spirit of DFT: 'exact' functional to be used for mean-field-type calculations

- Thus, this functional must produce a static RPA response function which is the 'exact' zero-energy response function.

  The RPA static polarizability should be regarded as the 'exact' one.

- Any modification of the response function (to go beyond the mean field) should be zero in the static limit to avoid double counting of correlations

- **It is required that the static polarizability is the same as in RPA (it can be demonstrated that the stability matrix is positive semi-definite):**

$$\Pi^{SRPA}(0) = \Pi^{RPA}(0) = -2m_{-1}^{RPA}$$

Inverse energy-weighted moment of the strength distribution conserved

$$\alpha^{RPA} = -\Pi(0) = 2\sum_{\nu} \frac{|<\nu|F|0>|^2}{E_\nu - E_0} = 2m_{-1}^{RPA}$$

Adachi, Lipparini, NPA 489, 445 (1988)

For cases where the interaction is density independent and A_{22} diagonal the expressions are simplified

$$\Omega^{SRPA} = \begin{bmatrix} A_{11'}(\omega) & B_{11'} \\ -B^{*}_{11'} & -A^{*}_{11'}(\omega) \end{bmatrix} \qquad \Omega^{RPA} = \begin{bmatrix} A_{11'} & B_{11'} \\ -B^{*}_{11'} & -A^{*}_{11'} \end{bmatrix}$$

where the energy-dependent matrix elements are

$$A_{11'}(\omega) = A_{11'} + \boxed{\sum_{2} A_{12}(\omega + i\eta - A_{22})^{-1} A_{2\,1'}}$$

Gambacurta, Grasso, Engel, PRC 92, 034303 (2015)

Energy-dependent self-energy insertion Σ(ω) -> leads to a beyond mean-field model and provides the description of spreading widths and fragmentation (in addition to the single-particle Landau damping) through de coupling with 2p2h

Tselyaev prescription to conserve the static polarizability: subtracting the self-energy calculated at zero energy to the energy-dependent self-energy Σ(E) - Σ(0)

Another way: correlated ground state (Papakonstantinou 2014, Takayanagi 1988)

RPA response function:

$$\Pi(E)=\begin{pmatrix} E-A & -B \\ -B & -E-A \end{pmatrix}^{-1}$$

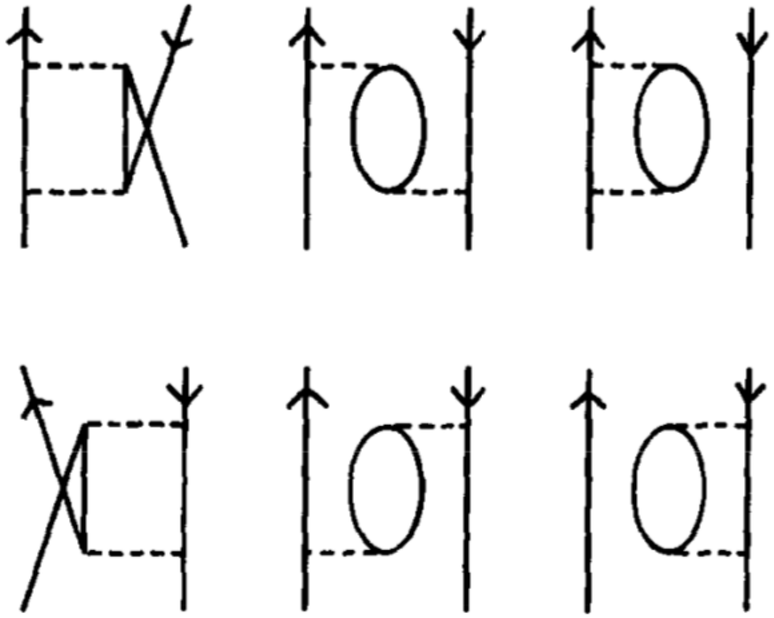
SRPA response function

$$\Pi(E)=\frac{1}{\begin{pmatrix} E-A & -B \\ -B & -E-A \end{pmatrix}-\begin{pmatrix} \Sigma(E) & \\ & \Sigma(-E) \end{pmatrix}}$$

SRPA diagrammatic representation of the self-energy (new diagrams included going from RPA to SRPA)

Response function calculated exactly up to second order of the residual interaction (new terms added coming from ground-state correlations)

$$\Pi(E)=\frac{1}{\begin{pmatrix} E-A & -B \\ -B & -E-A \end{pmatrix}-\begin{pmatrix} \Sigma(E)+\boxed{\delta\Sigma(E)} & \delta B \\ \delta B & \Sigma(-E)+\delta\Sigma(-E) \end{pmatrix}}$$



SOME RECENT APPLICATIONS

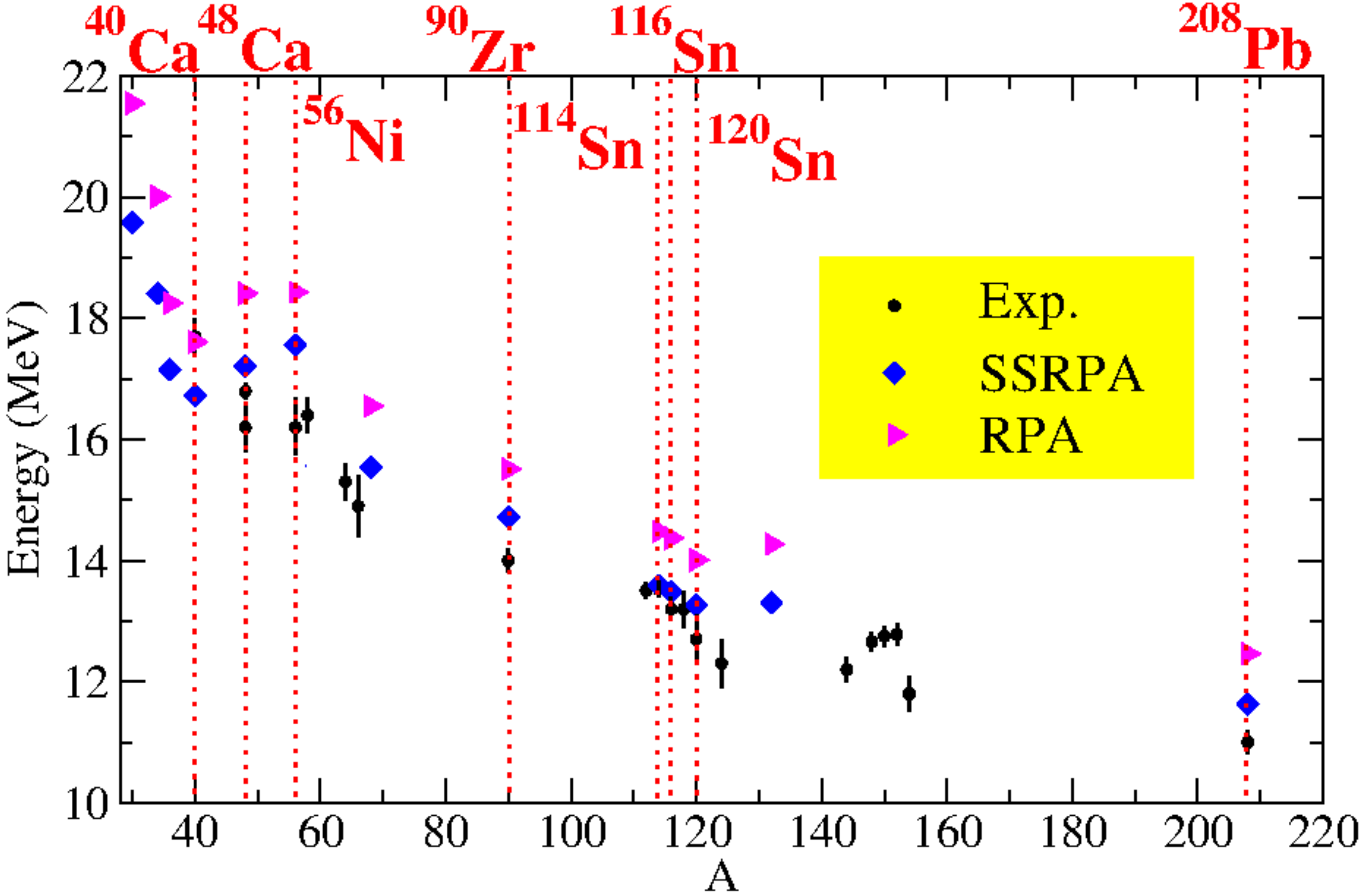
◆ **Systematic study of axial compression modes**
Vasseur, Gambacurta, Grasso, Phys Rev C 98, 044313 (2018)

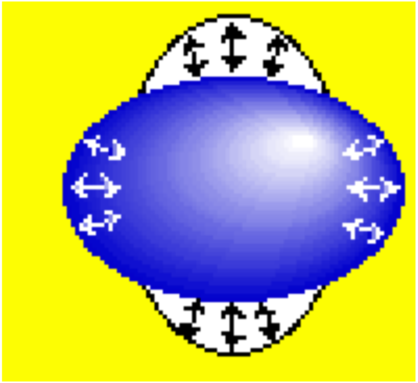
◆ **Beyond-mean-field effects on effective masses**
Grasso, Gambacurta, Vasseur, Phys Rev C 98, 051303(R) (2018)

◆ **Beyond-mean-field effects on the symmetry energy and its density dependence**
Grasso, Gambacurta, submitted Phys Rev C

◆ **Soft compression modes and compressibility**
Gambacurta, Grasso, Sorlin, Phys Rev C 100, 014317 (2019)

Isoscalar GQRs (axial compression or breathing modes) from ^{30}Si to ^{208}Pb
Centroids (folding with narrow Lorentzian distributions and adjusting on it a Lorentzian distribution)

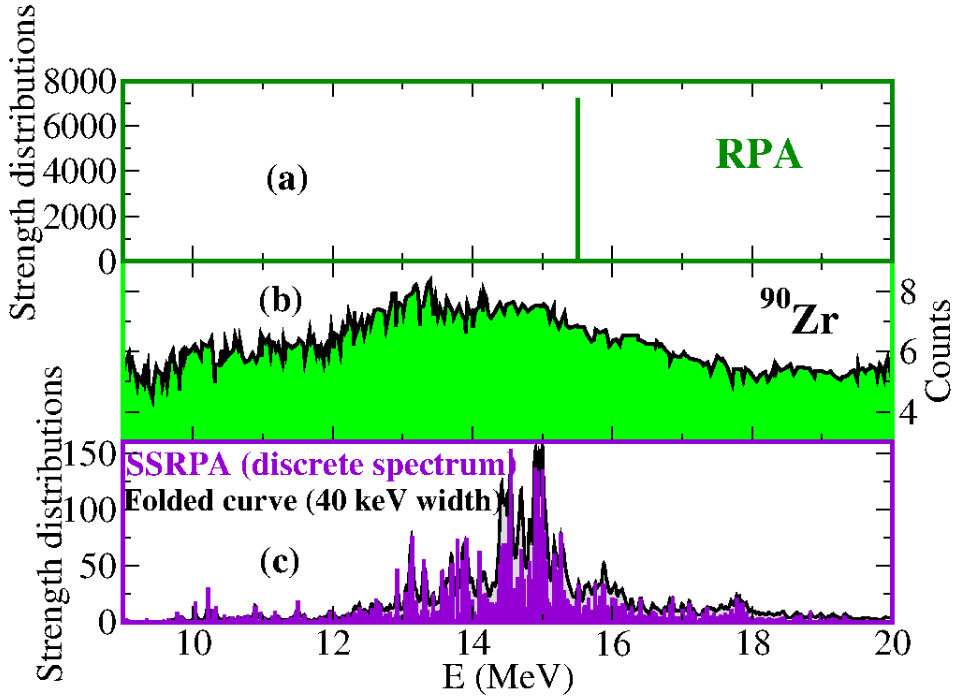




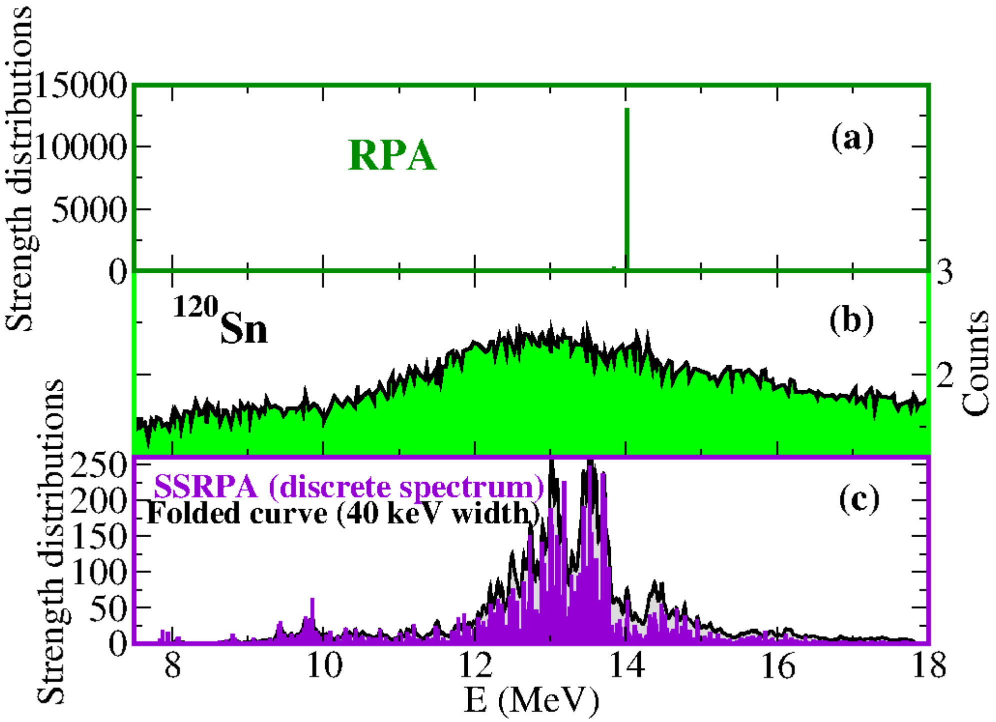
Globally: better agreement with the experimental data compared to RPA

Vasseur, Gambacurta, Grasso, PRC 98, 044313 (2018)

High-resolution proton inelastic scattering (p,p') spectra measured at iThemba LABS



SSRPA: discrete spectra and folded spectra with a Lorentzian of width equal to 40 keV (equal to the experimental energy resolution)



Exp. data:
Shevchenko et al, PRL 93, 122501 (2004)

Vasseur, Gambacurta, Grasso, PRC 98, 044313 (2018)

Effective masses m*/m

- Landau's theory of Fermi liquids: the system of interacting particles is described through quasiparticles having an effective mass m*

- Study of m* (relevant for the properties related to the propagation of particles in a medium): broad interest in many-body physics. Impact on, for example:

- Density of states in a many-body system
- Specific heat of a low-temperature Fermi gas
- Maximum mass of a neutron star
- Energies of axial compression or breathing modes in atomic gases and in nuclei (Isoscalar Giant Quadropole Resonances)

Effective masses in Fermi liquids

First dynamic measurement of the polaron effective mass

PRL 103, 170402 (2009) PHYSICAL REVIEW LETTERS week ending 23 OCTOBER 2009

Collective Oscillations of an Imbalanced Fermi Gas: Axial Compression Modes and Polaron Effective Mass

S. Nascimbène, N. Navon, K. J. Jiang, L. Tarruell,* M. Teichmann,† J. McKeever,‡ F. Chevy, and C. Salomon
Laboratoire Kastler Brossel, CNRS, UPMC, École Normale Supérieure, 24 rue Lhomond, 75231 Paris, France

The Fermi polaron is an impurity immersed in a Fermi sea (strongly imbalanced Fermi gases) *.

Based on the Landau theory of Fermi liquids, the energy spectrum of the polaron is similar to that of a free particle. Using the local-density approximation, the frequency ω* of the polaron is

$$\frac{\omega^*}{\omega} = \sqrt{\frac{1-A}{m^*/m}}.$$

ω is the frequency of the trap (harmonic oscillator), A is a dimensionless quantity that characterizes the attraction of the impurity by the other atoms

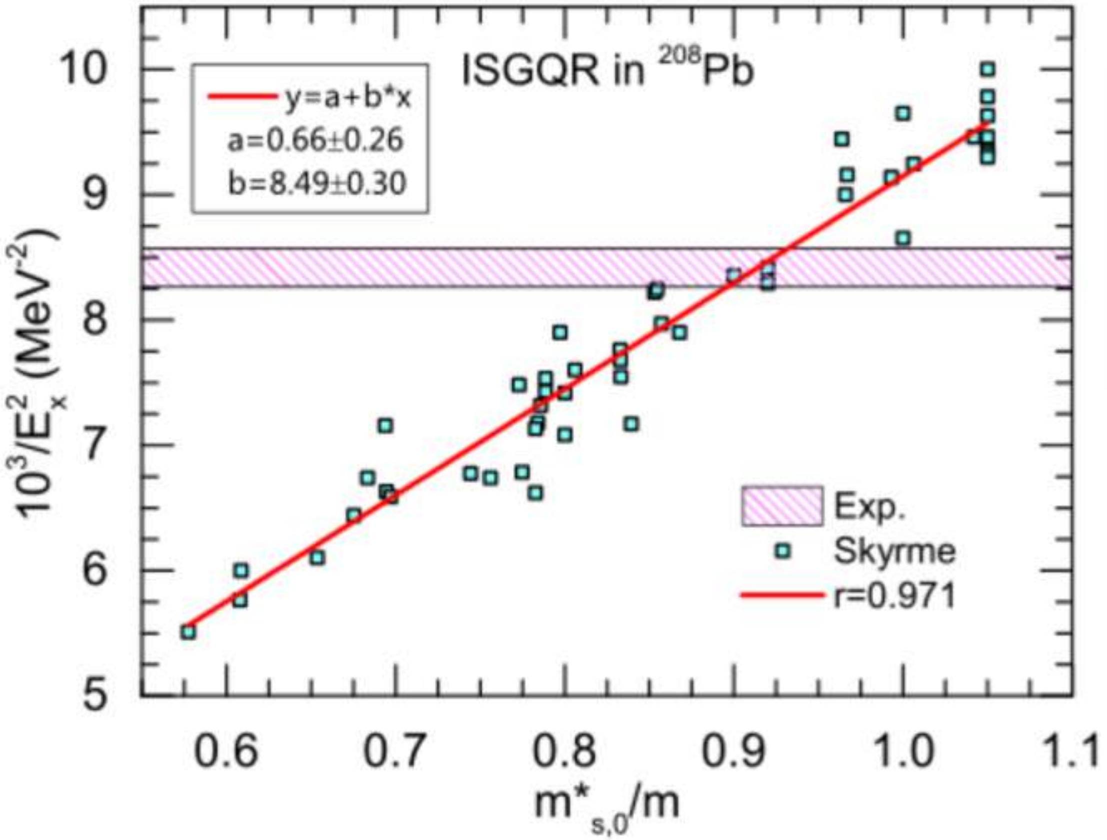
Lobo, Recati, Giorgini, Stringari, PRL 97, 200404 (2006)

* Analogous calculations for nuclear systems:
Forbes et al., PRC 89, 041301 (R) (2014)
Roggero et al., PRC 92, 054303 (2015)

Effective masses in Fermi liquids

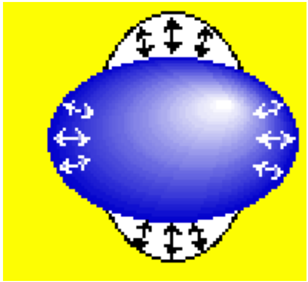
The axial breathing mode in nuclear physics corresponds to the isoscalar GQR.

Based on the Landau theory of Fermi liquids, relation between the centroid energy of the IS GQR and $(m/m^*)^{1/2}$ known and used

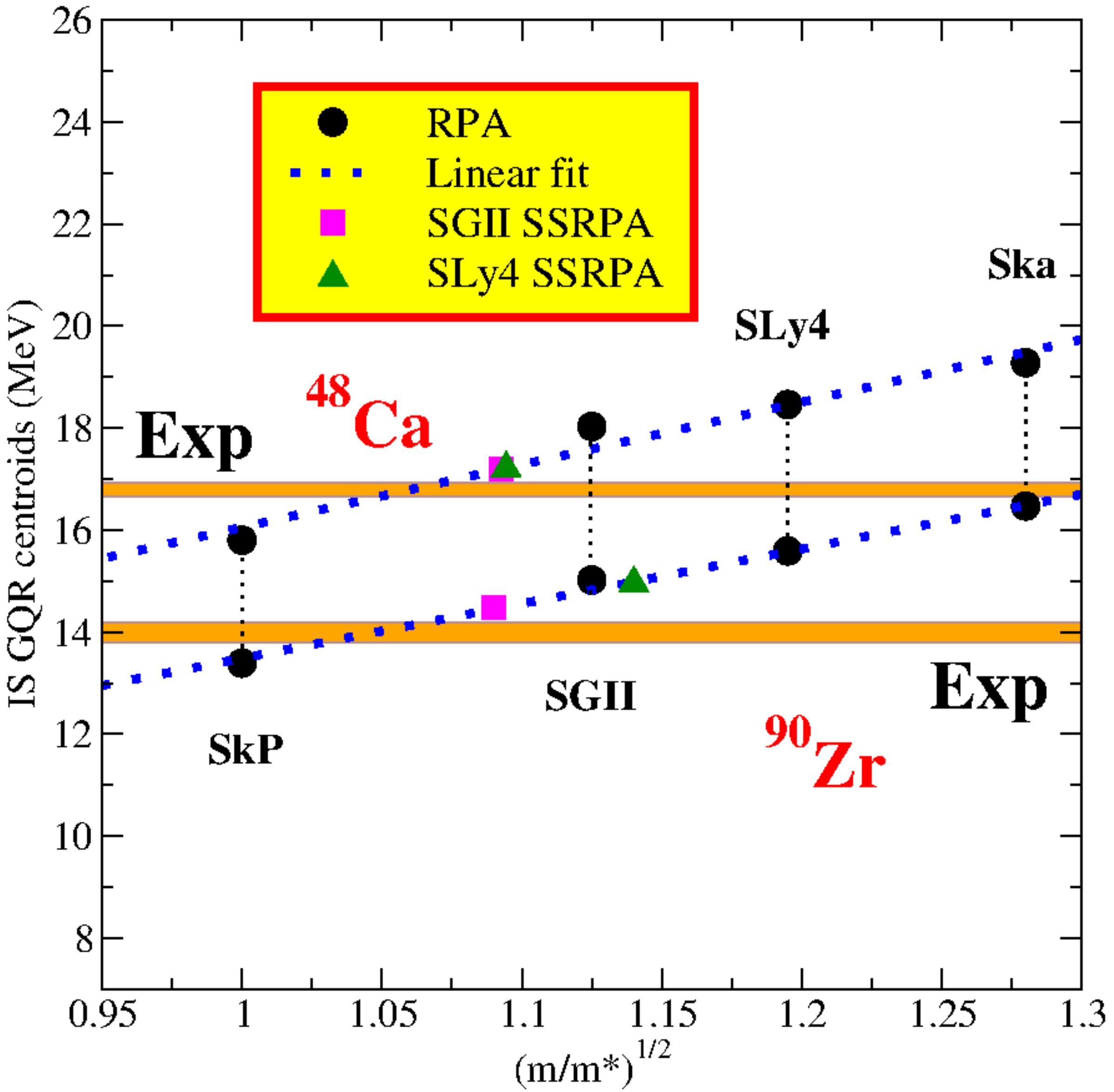


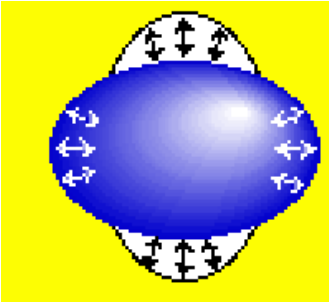
Bao-An Li et al., Prog. Part. Nucl. Phys. 99, 29 (2018)

Blaizot, Phys. Rep. 64, 171 (1980)



Beyond-mean-field (SSRPA) effective masses in the nuclear Fermi liquid from axial breathing modes





Grasso, Gambacurta, Vasseur,
PRC 98, 051303(R) (2018)

SSRPA extraction of the effective mass

Definition of effective mass:

$$\frac{1}{m^*} = \frac{dE}{dk}\frac{1}{\hbar^2 k}$$

for a particle of energy E and momentum k, with

$$E = \frac{\hbar^2 k^2}{2m} + \Sigma_k + \Sigma_{k,E}.$$

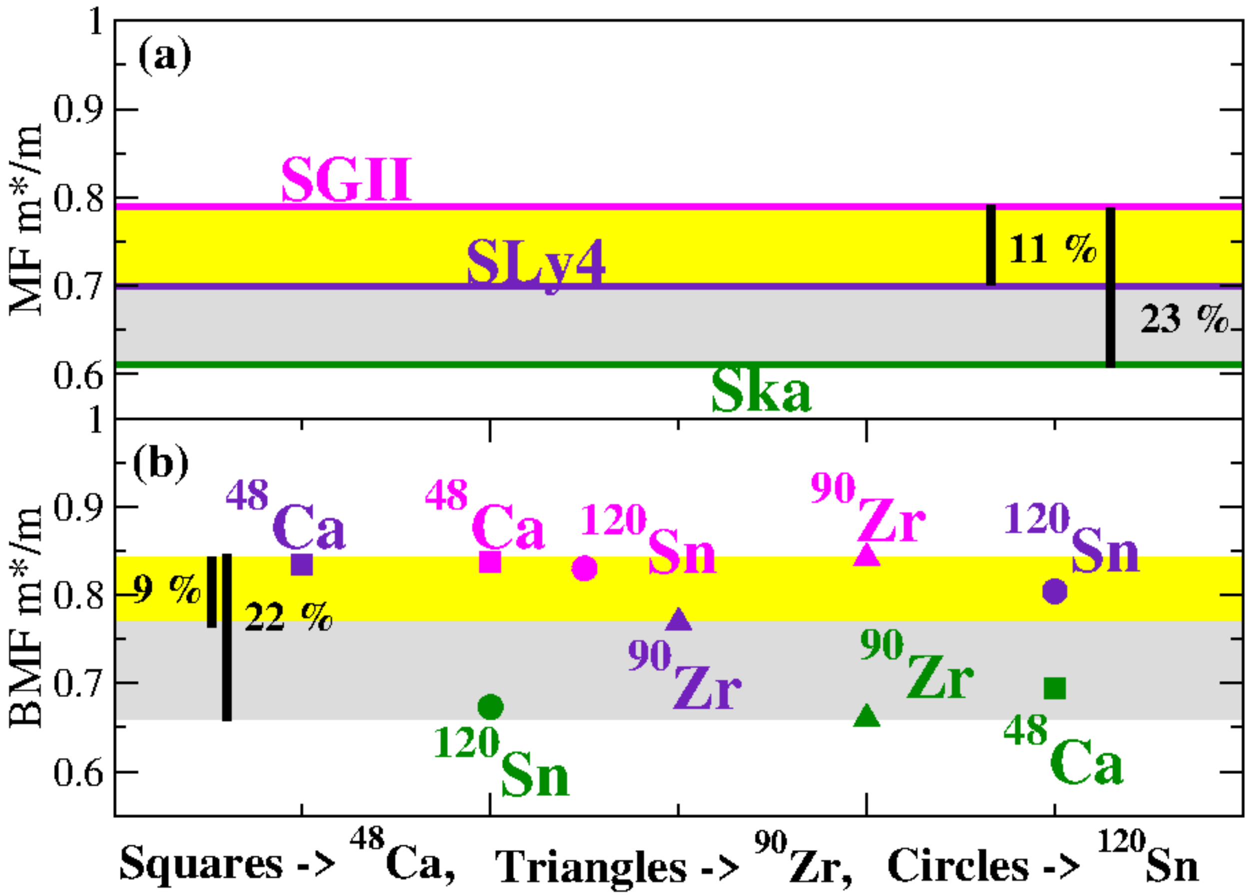
k-mass (leading order of the Dyson equation and E-mass -> beyond mean field, energy dependence of the self-energy)

$$\begin{aligned}\frac{m^*}{m} &= \left(1 - \frac{\partial \Sigma_{k,E}}{\partial E}\right) \cdot \left(1 + \frac{m}{\hbar^2 k}\frac{\partial \Sigma_k}{\partial k}\right)^{-1} \\ &\equiv \frac{m_E^*}{m} \cdot \frac{m_k^*}{m},\end{aligned}$$

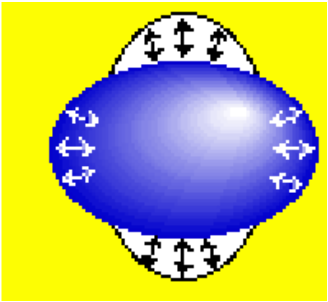
One may extract, for each nucleus and for each interaction, an estimation of the E-mass (equal to 1 at the mean-field level).

We have found an enhancement of the E-mass between 6 and 16% with SSRPA (nucleus and interaction dependence)

Beyond-mean-field effective masses. Enhancement

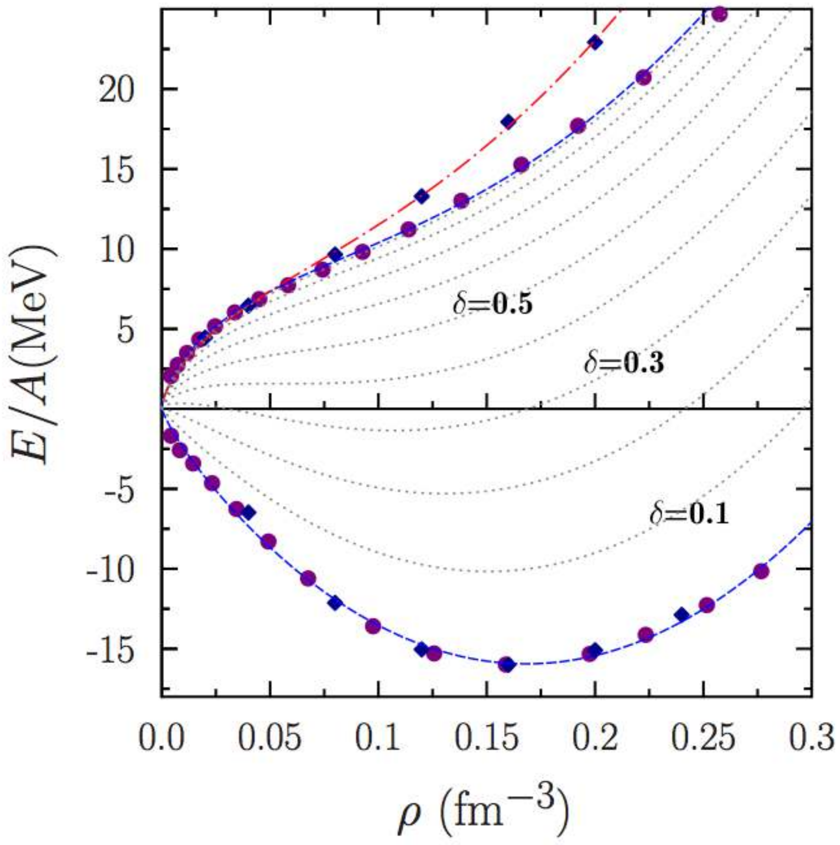


Grasso, Gambacurta, Vasseur,
PRC 98, 051303(R) (2018)



Mean field -> dispersion related to the used interaction
Beyond-mean-field -> in addition, nucleus dependence. However, theoretical error not larger than for the mean-field case

Beyond-mean-field effects on the symmetry energy and its density dependence (slope) deduced from the low-energy dipole response in ^{68}Ni



Equations of state (EOS) of pure neutron (top) and symmetric (bottom) infinite nuclear matter

EOS for asymmetric matter:

$$E(\rho, \delta) = E(\rho, \delta = 0) + S(\rho)\delta^2 + \mathcal{O}(\delta^4)$$

$$(\rho_n - \rho_p)/\rho$$

Symmetry-energy coefficient:

$$S(\rho) = \frac{1}{2}\frac{\partial^2}{\partial\delta^2}E(\rho, \delta)|_{\delta=0}$$

$$\boxed{J = S(\rho_0)}$$

Symmetry-energy coeff. computed at the saturation density of symmetric matter

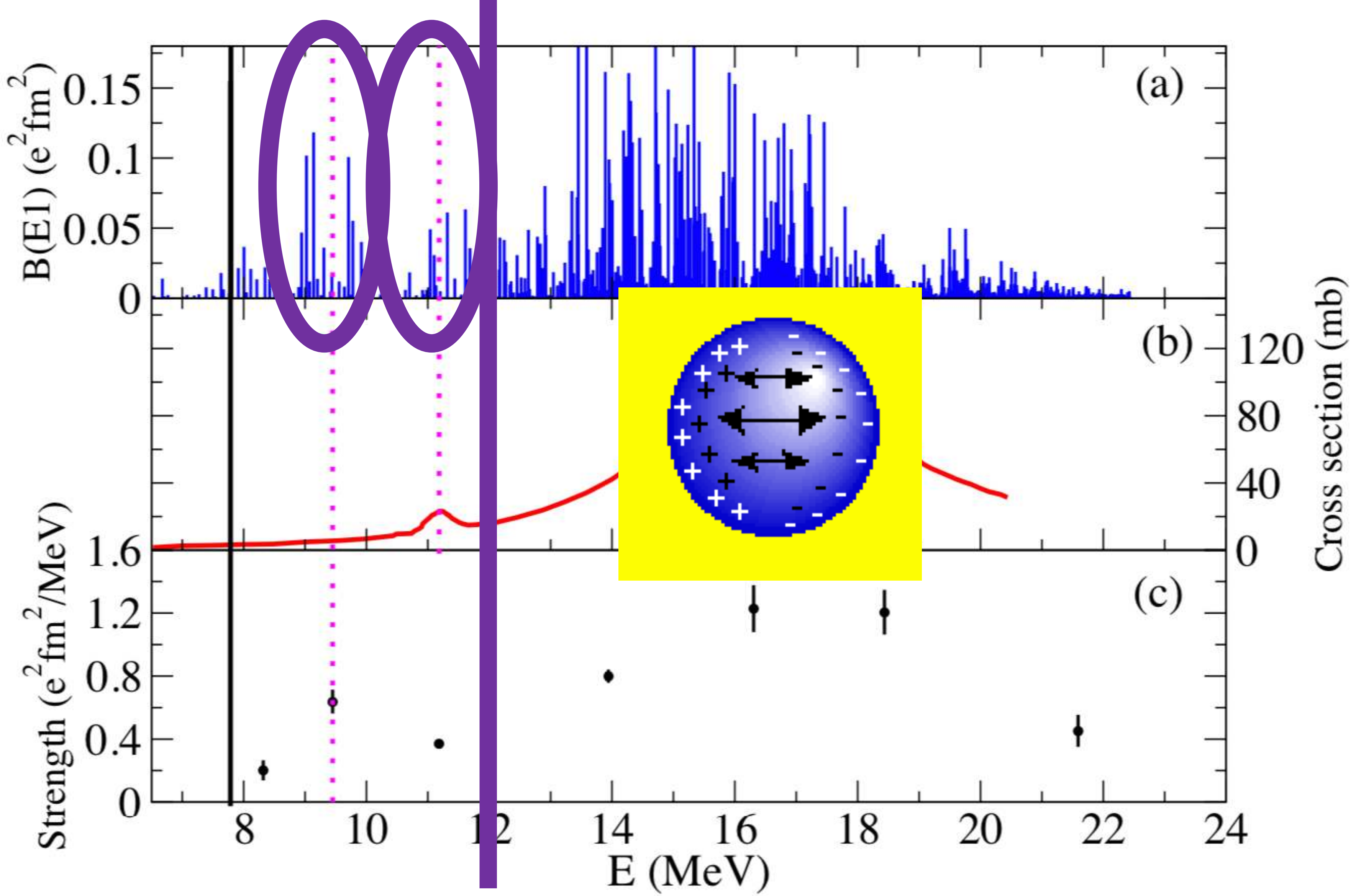
Expanding S around the saturation density:

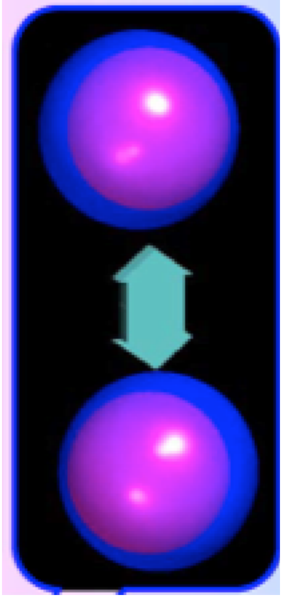
$$S(\rho) = J + L\frac{\rho - \rho_0}{3\rho_0} + \frac{1}{2}k_{sym}(\frac{\rho - \rho_0}{3\rho_0})^2 + \mathcal{O}[(\rho - \rho_0)^3]$$

$$\boxed{L = 3\rho_0\frac{\partial S(\rho)}{\partial\rho}|_{\rho=\rho_0}}$$

L -> slope of the symmetry energy

Dipole response in ^{68}Ni (isospin-asymmetric nucleus -> neutron skin)





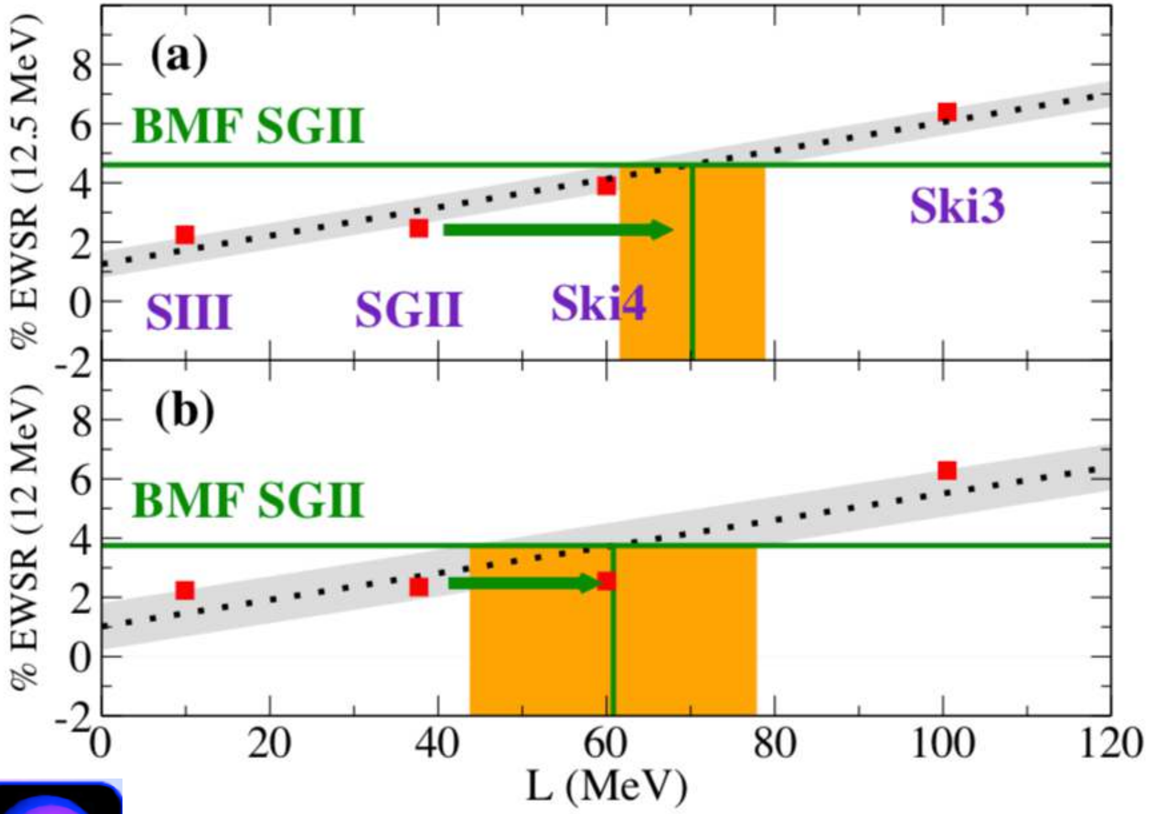
Oscillations of the neutron skin against the core of the nucleus

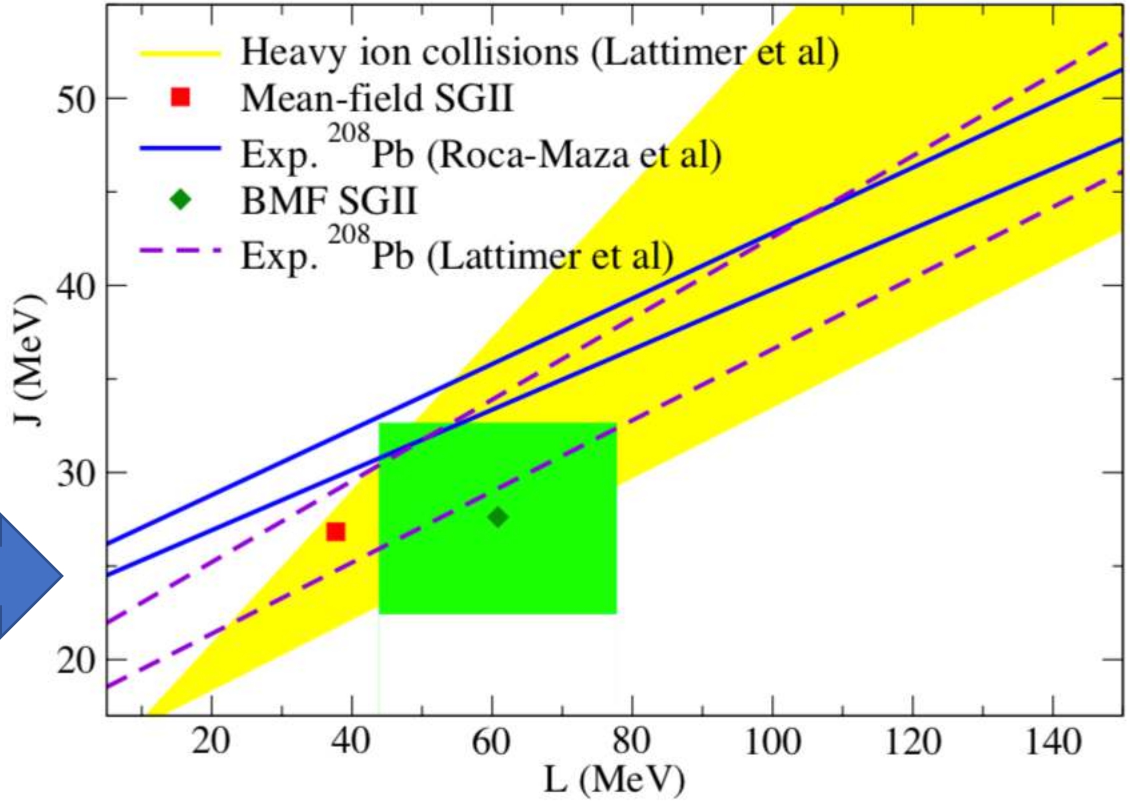
(b): photoabsorption cross section from Wieland et al, PRL 102, 092502 (2009)

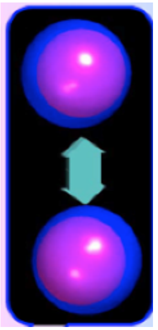
(c) : E1 strength distribution from Rossi et al, PRL 111, 242503 (2013)

Grasso, Gambacurta, submitted PRC

Beyond-mean-field effects on the symmetry energy and its density dependence (slope) deduced from the low-energy dipole response in ^{68}Ni





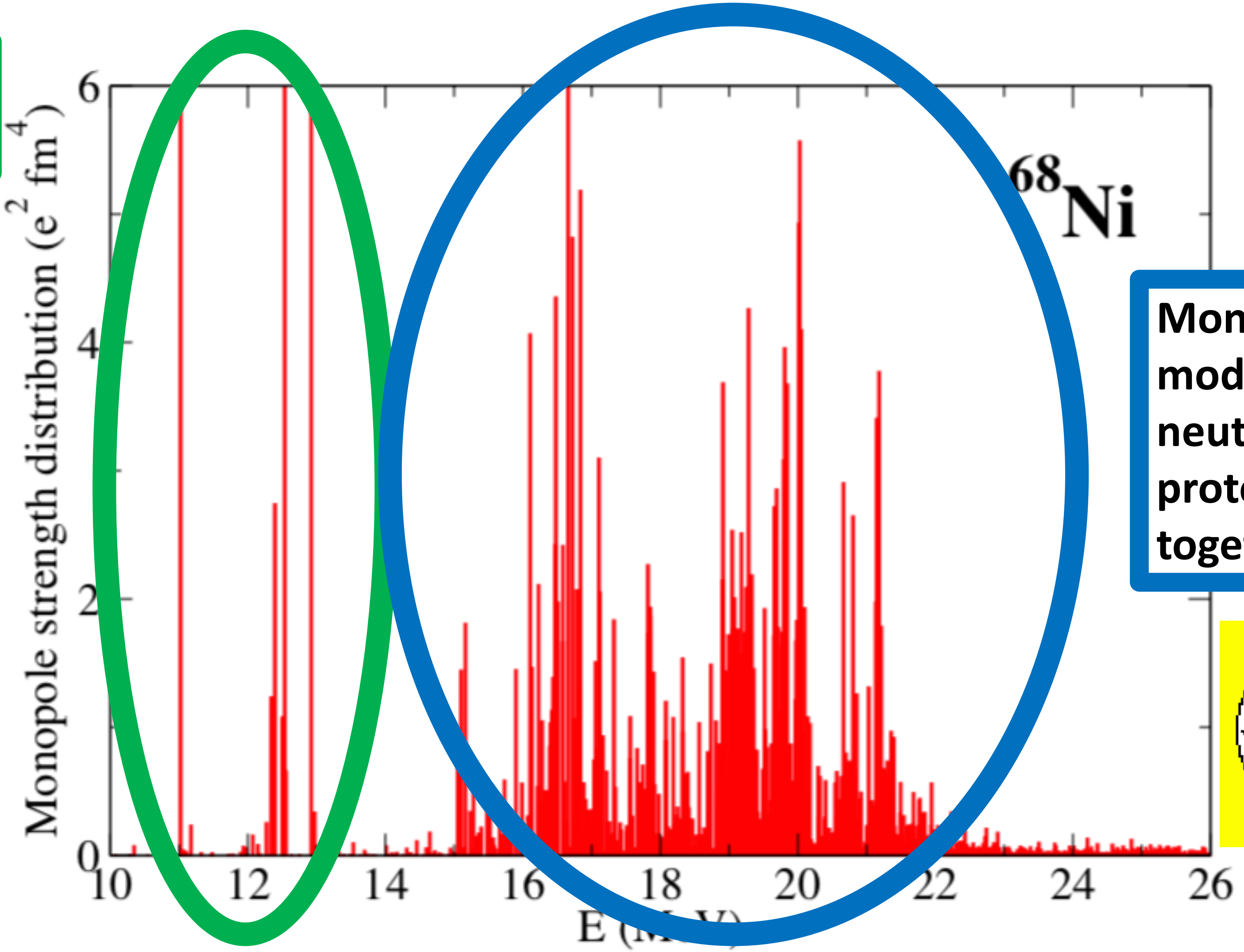


Grasso, Gambacurta,
Submitted PRC

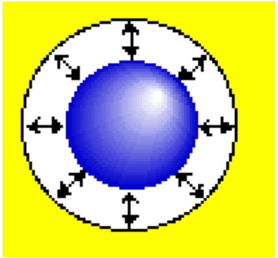
	L (MeV)	Δr_{np} (fm)	J (MeV)
Mean field	37.70	0.154	26.83
BMF	60.815 ± 16.982	0.173 ± 0.018	27.617 ± 5.004

Low-energy compression modes with the same isospin-asymmetric nucleus, ^{68}Ni (not yet measured)

Low-energy region,
Soft modes



Monopole breathing mode where neutrons and protons move together coherently



Gambacurta, Grasso, Sorlin, PRC 100, 014317 (2019)

Link between the excitation where neutrons and protons move coherently together (giant resonance) with the compressibility (curvature of the EOS of symmetric matter)

For infinite symmetric matter:

$$K = 9\rho_0^2\left(\frac{\partial^2 E^{sym}/A}{\partial\rho^2}\right)_{\rho=\rho_0}$$

K -> compressibility
ρ_0 -> equilibrium density of symmetric matter (saturation density)

By treating the nucleus as a liquid drop, starting from the hydrodynamics equations, one may write a relation between the centroid energy of the compression mode and the compressibility

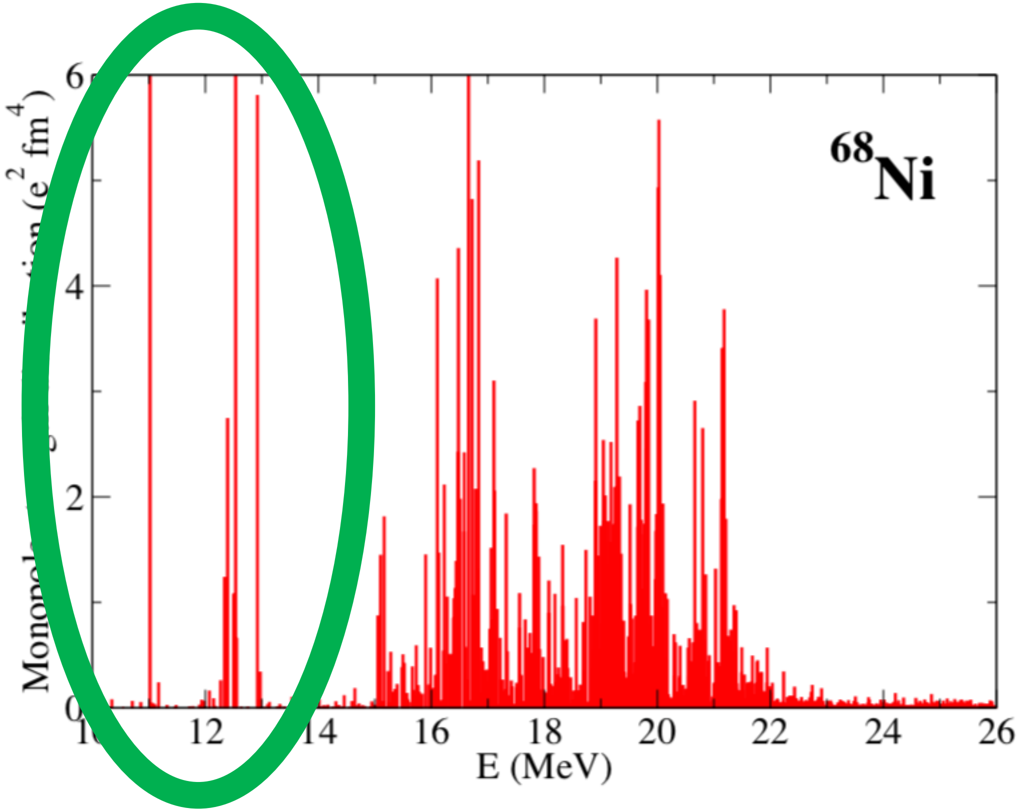
$$E = \sqrt{\frac{\hbar^2\pi^2}{15m}}\sqrt{\frac{K}{\eta_0^2}}$$

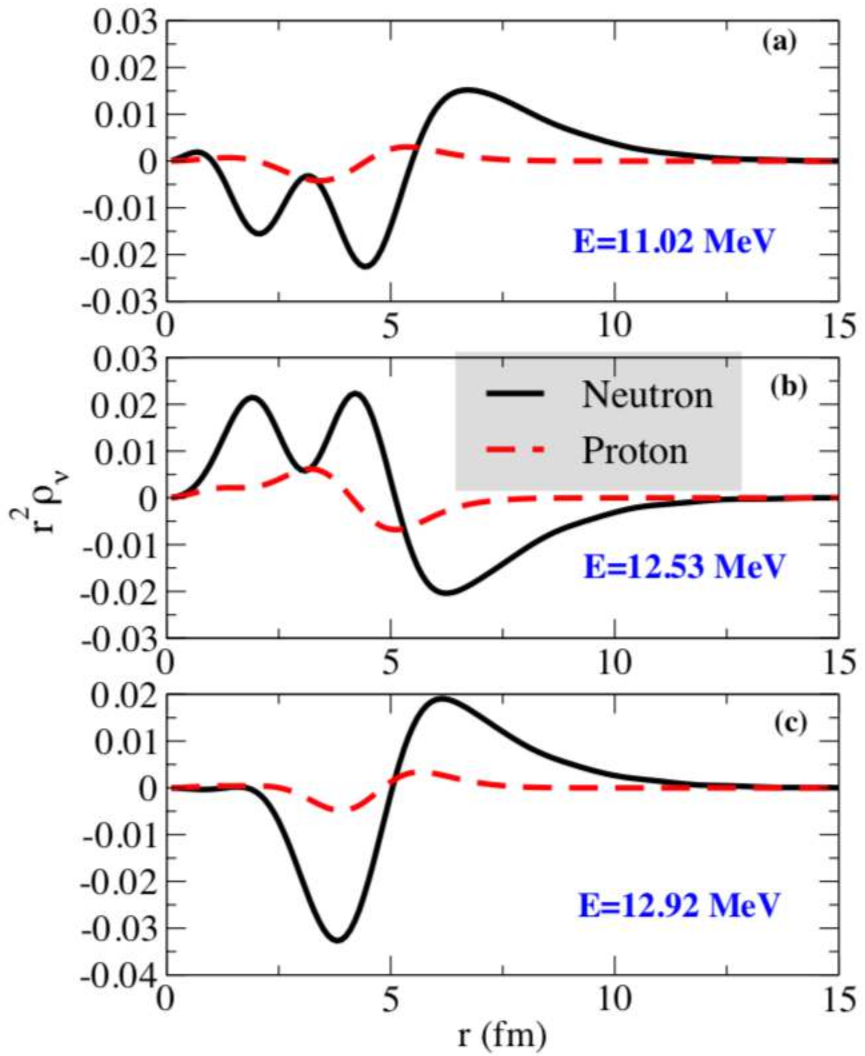
Root mean square radius

$$\eta_0 = r_0 A^{1/3}, \text{ with } r_0 \sim 1 \text{ fm}$$

$$E \sim 5.22 A^{-1/3}\sqrt{K}$$

Low-energy compression (monopole) modes. Transition densities





Neutron excitations

Gambacurta, Grasso, Sorlin, PRC 100, 014317 (2019)

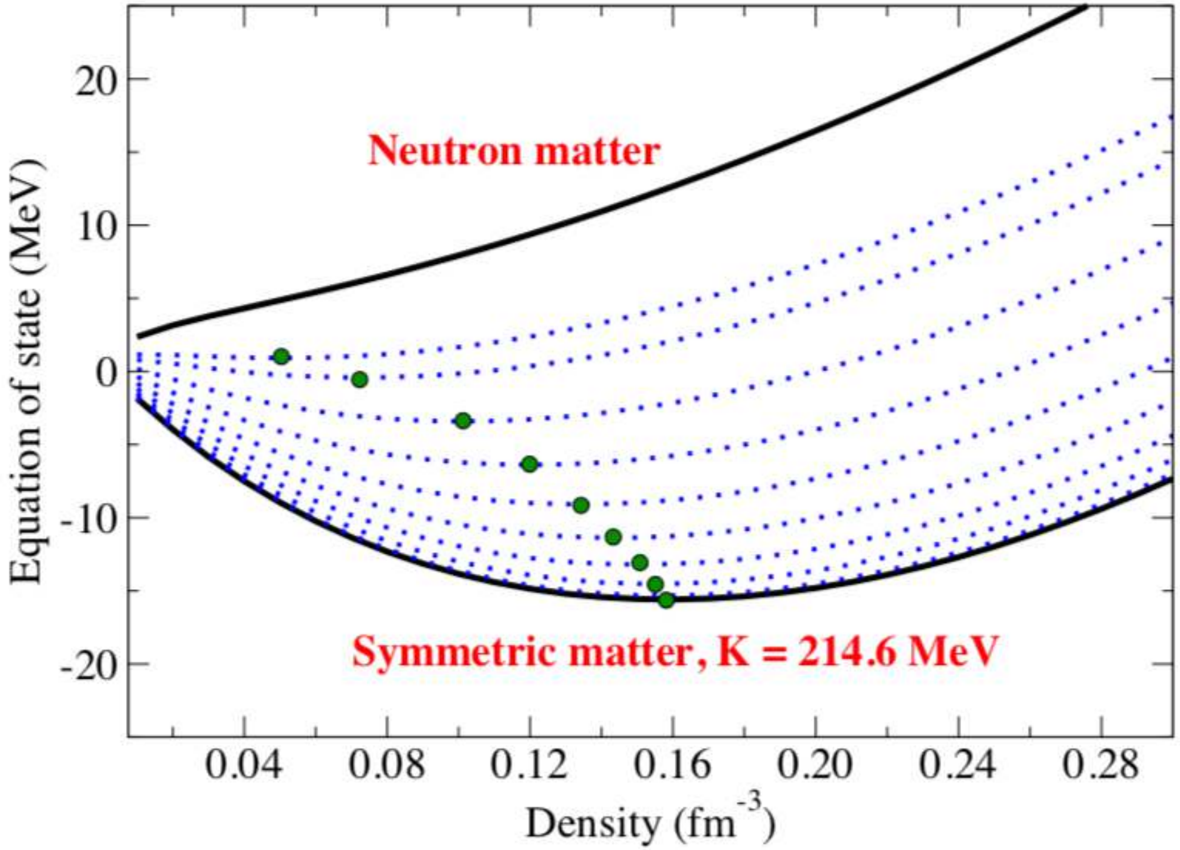
Link with compressibility? Soft breathing modes in neutron-rich systems are strongly driven by neutrons

Link with a compressibility defined for asymmetric (neutron-rich) matter ?

$$K_X = 9\rho_{eq}^2 \left(\frac{\partial^2 E^X/A}{\partial \rho^2} \right)_{\rho=\rho_{eq}}$$

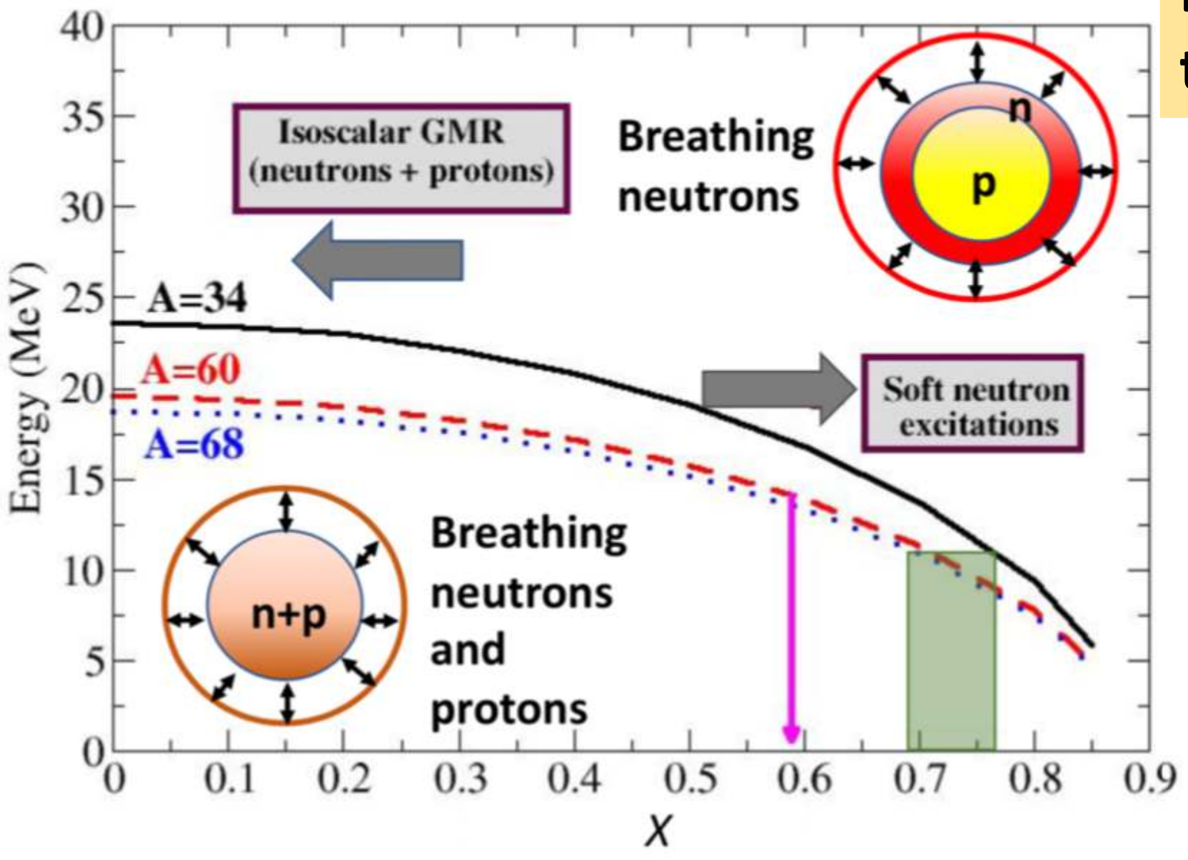
$$E(X) \sim 5.22 A^{-1/3} \sqrt{K_X}$$

X -> isospin asymmetry of asymmetric matter (and of the oscillating system). (N-Z)/A



Gambacurta, Grasso, Sorlin, PRC 100, 014317 (2019)

Plotting $E(X) \sim 5.22 A^{-1/3} \sqrt{K_X}$ for a given A



Gambacurta, Grasso, Sorlin, PRC 100, 014317 (2019)

Estimating the isospin asymmetry of the oscillating system

Green area: range of isospin asymmmetry for the oscillating system involved in the excitations at 8.8 (^{60}Ca), 11.1 (^{34}Si), 11.0 (^{68}Ni) MeV.

Magenta arrow: same, for the excitation at 14.1 MeV in ^{60}Ca.

Summary

- Implementation of the SRPA model by a subtraction procedure: double counting, stability condition (shift correction with respect to RPA), convergence with respect to the cutoff

- Some recent applications:

◆ Systematic study of GQRs (compared to RPA: centroids globally in better agreement with the experimental data; enhancement of the widths owing to the description of the spreading width)

◆ Beyond-mean-field effect on the effective mass (extraction of an enhanced effective mass produced by beyond-mean-field effects)

◆ Beyond-mean-field effect on the symmetry energy and its slope (deduced from the low-energy dipole response)

◆ Low-lying compression modes in neutron-rich nuclei (neutron-driven modes. Link with the compressibility defined for asymmetric infinite matter)

- Work in progress: describing superfluidity effects (Cooper pairs)

By following Tselyaev 2013 ->

It is possible to rewrite the equations (after subtraction) in a non energy dependent SRPA form:

$$\mathcal{A}_F^S = \begin{pmatrix} A_{11'} + \boxed{\sum_2 A_{12}(A_{22'})^{-1}A_{21'} + \sum_2 B_{12}(A_{22'})^{-1}B_{21'}} & A_{12} \\ A_{21} & A_{22'} \end{pmatrix}$$

$$\mathcal{B}_F^S = \begin{pmatrix} B_{11'} + \sum_2 A_{12}(A_{22'})^{-1}B_{21'} + \sum_2 B_{12}(A_{22'})^{-1}A_{21'} & B_{12} \\ B_{21} & B_{22'} \end{pmatrix}$$

S -> subtracted
F -> full scheme (inversion of the matrix $A_{22'}$)

Gambacurta, Grasso, Engel, PRC 92, 034303 (2015)

RPA versus SRPA

- Tselyaev, PRC 88, 054301 (2013) (subtraction)
- Papakonstantinou, PRC 90, 024305 (2014) (correlated ground state)

- Stability Condition (Thouless 1960)

HF state minimizes the expectation value of the Hamiltonian -> RPA stability matrix ipositive semi-definite (real eigenvalues; eigenvectors with positive eigenvalues have positive norm)

$$S^{RPA} = M^{RPA}\Omega^{RPA} = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}$$

$$M^{RPA} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Thouless theorem (Thouless 1961) (restoration of symmetries in self-consistent calculations: spurious excitations such as center of mass translation located at zero excitation energy)

It was demonstrated (Yannouleas 1987) that the energy weighted sum rule (EWSR) is the same in RPA and in SRPA for any single-particle operator (Thouless 1961)..... BUT ...

- HF state minimizes the expectation value of the Hamiltonian -> does not guarantee that the SRPA stability matrix is positive semi-definite (instabilities and unphysical shifts of the spectrum)

In the EDF scheme for beyond mean field:
- Ultraviolet divergence for zero-range interactions (dependence on the energy cutoff for the 2p2h configurations)
– Overcounting of correlations (if parameters of the interactions are adjusted at the mean-field level)

Stability condition in RPA (Thouless theorem, Nucl. Phys. 21, 225 (1960), Nucl. Phys. 22, 78 (1961))

If the HF state minimizes the expectation value of the Hamiltonian -> the RPA stability matrix is positive semi-definite (real eigenvalues; eigenvectors with positive eigenvalues have positive norm)

Stability RPA matrix

$$S^{RPA} = M^{RPA}\Omega^{RPA} = \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix}$$

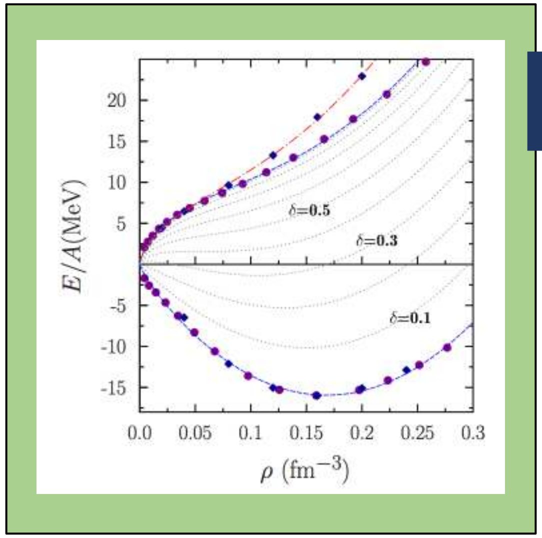
$$M^{RPA} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This does not imply that the SRPA stability matrix is also positive semi-definite.

The theorem can be extended to extensions of RPA by applying the subtraction procedure (Tselyaev 2013)

Equation of state of infinite matter.

For example, links with collective modes



- Incompressibility (ISGMR, ISGDR)
- Effective mass (axial compression modes -> ISG R)
- Symmetry energy and its density dependence, neutron skin (IVGDR, dipole polarizability, pygmy dipole, IVGQR, IAS, Gamow-Teller, spin dipole, anti-analogue GDR)

- Grasso, Effective density functionals beyond mean field, Prog. Part. Nucl. Phys. 106, 256 (2019)
- Bao-An Li, Bao-Jun Cai, Lie-wen Chen, and Jun Xu, Nucleon effective masses in neutron-rich matter, Prog. Part. Nucl. Phys. 99, 29 (2018)
- Roca-Maza and Paar, Nuclear equation of state from ground and collective excited state properties of nuclei, Prog. Part. Nucl. Phys. 101, 96 (2018)
- Garg, Colò, The compression-mode giant resonances and nuclear incompressibility, Prog. Part. Nucl. Phys. 101, 55 (2018)
- Bracco, Lanza, Tamii, Isoscalar and isovector dipole ..., Prog. Part. Nucl. Phys. 106, 360 (2019)

If the interaction does not depend on the density

- $B_{12} = B_{21} = B_{22} = 0$
- The beyond-RPA matrix elements for the matrix A are:

Coupling 1p1h with 2p2h (matrix elements of the interaction: hppp, phhh)

$$
\begin{aligned}
A_{12} &= A_{ph,p_1p_2h_1h_2} \\
&= \langle \mathrm{HF}|[a_h^\dagger a_p, [H, a_{p_1}^\dagger a_{p_2}^\dagger a_{h_2} a_{h_1}]]|\mathrm{HF}\rangle \\
&= \chi(h_1, h_2)\bar{V}_{h_1pp_1p_2}\delta_{hh_2} - \chi(p_1, p_2)\bar{V}_{h_1h_1p_1h}\delta_{pp_2},
\end{aligned}
$$

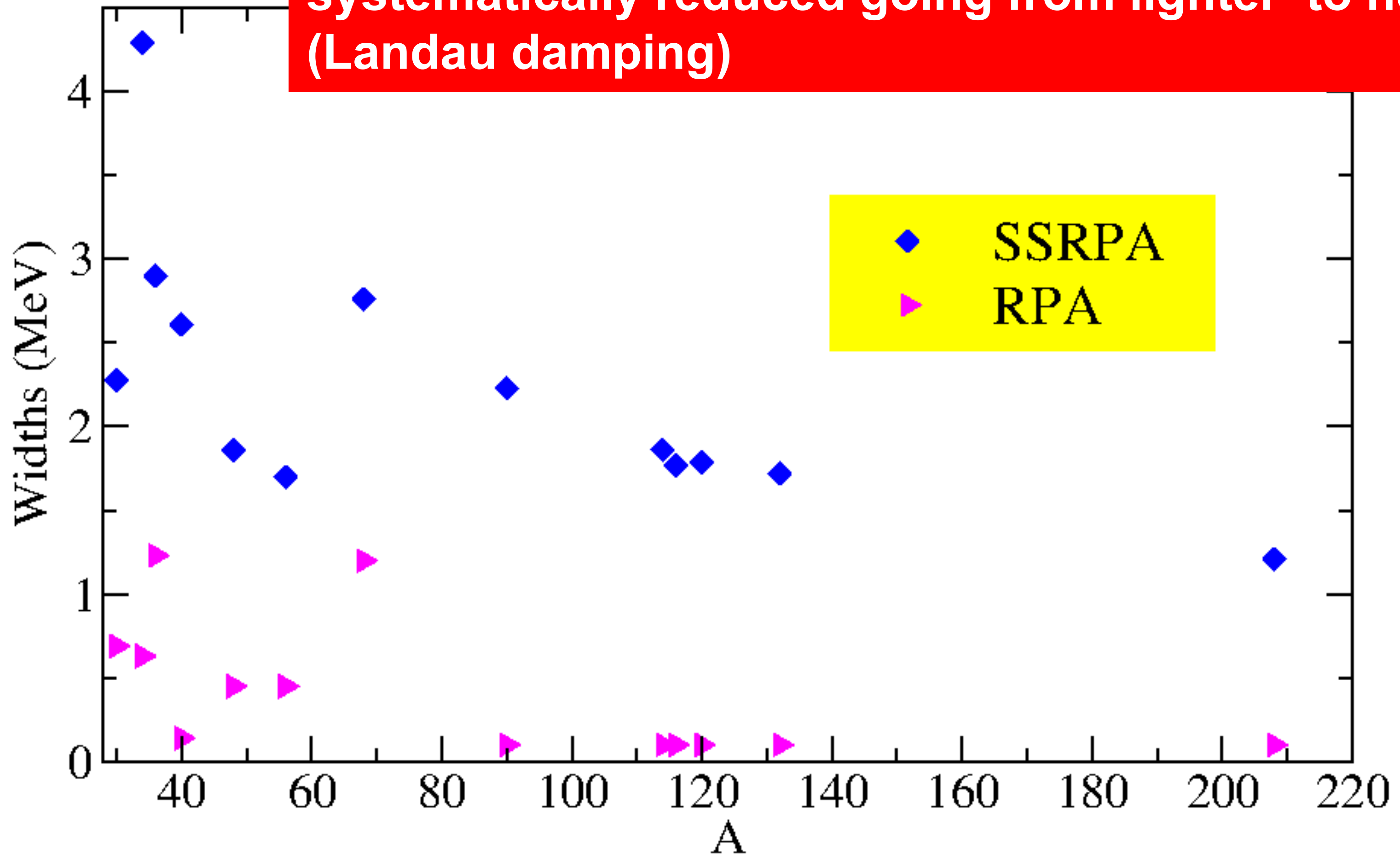
Antisymmetrizer

Coupling 2p2h with 2p2h (matrix elements of the interaction: pppp, hhhh, phhp)

$$
\begin{aligned}
A_{22} &= A_{p_1h_1p_2h_2,p_1'h_1'p_2'h_2'} \\
&= \langle \mathrm{HF}|[a_{h_1}^\dagger a_{h_2}^\dagger a_{p_1} a_{p_2}, [H, a_{p_2'}^\dagger a_{p_1'}^\dagger a_{h_2'} a_{h_1'}]]|\mathrm{HF}\rangle \\
&= (\epsilon_{p_1} + \epsilon_{p_2} - \epsilon_{h_1} - \epsilon_{h_2})\chi(p_1, p_2)\chi(h_1, h_2) \\
&\quad \times \delta_{h_1h_1'}\delta_{p_1p_1'}\delta_{h_2h_2'}\delta_{p_2p_2'} + \chi(h_1, h_2)\bar{V}_{p_1p_2p_1'p_2'}\delta_{h_1h_1'}\delta_{h_2h_2'} \\
&\quad + \chi(p_1, p_2)\bar{V}_{h_1h_2h_1'h_2'}\delta_{p_1p_1'}\delta_{p_2p_2'} \\
&\quad + \chi(p_1, p_2)\chi(h_1, h_2)\chi(p_1', p_2')\chi(h_1', h_2') \\
&\quad \times \bar{V}_{p_1h_1'h_1p_1'}\delta_{h_2h_2'}\delta_{p_2p_2'},
\end{aligned}
$$

Isoscalar GQRs. Widths (folding with narrow Lorentzian distributions and adjusting on it a Lorentzian distribution)

General trend, found both in RPA and in SSRPA: the width is systematically reduced going from lighter to heavier nuclei (Landau damping)



Vasseur, Gambacurta, Grasso, PRC 98, 044313 (2018)